Equilibrium Problems

When the net force – the vector sum of all the forces – acting on a body is zero, the object is in **equilibrium**. According to Newton's 1st law, the object will not accelerate because there is no net force acting on it.

If the body is in motion with a constant velocity, then we say it is in **dynamic equilibrium**. If the velocity of the body is zero, then the body is at rest and is said to be in **static equilibrium**.

The condition of equilibrium for any object is given mathematically by

$$\overrightarrow{F_{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... = 0$$

This can also be written as

$$\overrightarrow{F_{net}} = \sum \vec{F} = 0$$

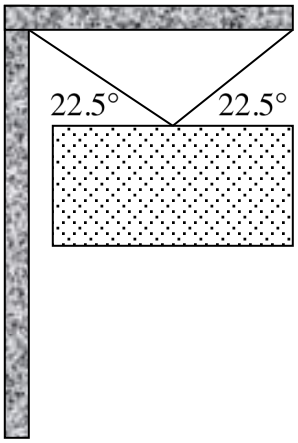
Where $\sum \vec{F}$ means the vector sum of all forces acting on the body.

Note: If $\sum \vec{F} = 0$, then the x and y components of $\sum \vec{F}$ must also each be zero. Thus, the equilibrium condition can be represented mathematically by:

$$\sum F_x = 0$$
$$\sum F_y = 0$$

Example 1

A sign that weighs $168\ N$ is supported by ropes a and b, as shown, that make $22.5°$ angles with the horizontal. The sign is not moving. What forces do the ropes exert on the sign?



Example 2

A child of mass $35\ kg$ sits in a light swing suspended by a rope of negligible mass. His sister pushes him forward by a horizontal force until the rope makes an angle of $30°$ with the vertical. What is the tension in the rope and how much horizontal force is required to hold the child in that position?

When the vector sum of forces acting on a body is not zero, a single force can be applied that will produce equilibrium. This force is called the **equilibrant force**. The magnitude of the equilibrant will equal the magnitude of the net force acting on the body. The direction of the equilibrant will be opposite to that of the net force.

Example 3
A force of $55\ N$ acts due west on an object. What added single force on the object produces equilibrium?

Example 4
Two forces act on an object. One force is $6.0\ N$ horizontally. The second force is $8.0\ N$ vertically. (a) Find the magnitude and direction of the resultant. (b) Find the magnitude and direction of the force that will produce equilibrium.

Homework
Equilibrium Worksheet

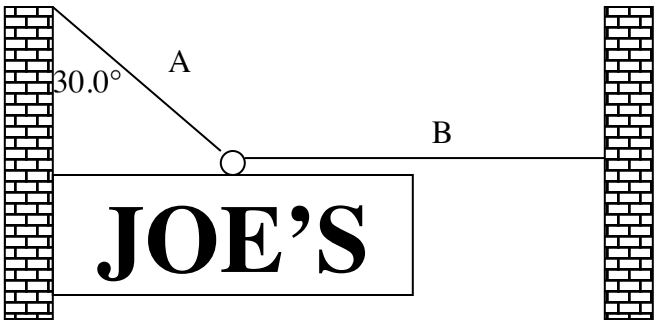
Equilibrium Worksheet

1. An object in equilibrium has three forces acting on it. A $33\ N$ force acts northward, and a $44\ N$ force acts $30°\ E\ of\ N$. What is the magnitude and direction of the third force? ($74\ N\ \left[17°\ W\ of\ S\right]$)

2. Five forces act on an object: the first, $60\ N\ \left[North\right]$; the second, $40\ N\ \left[East\right]$; the third, $80\ N\ \left[South\right]$; the fourth, $40\ N\ \left[West\right]$; and the fifth, $50\ N\ \left[30°\ E\ of\ N\right]$. What is the magnitude and direction of a sixth force that produces equilibrium of the object? ($34.2\ N\ \left[43°\ S\ of\ W\right]$)

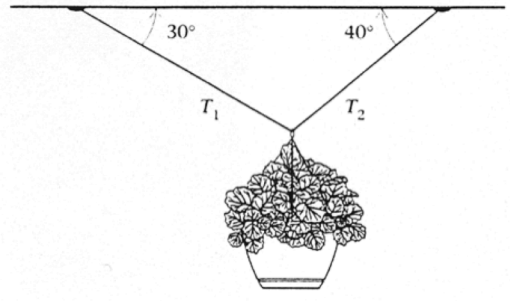
3. A street lamp weighs $150\ N$. It is supported equally by two wires that form an angle of 120° with each other.
   a. What is the tension of each of these wires? ($150\ N$)
   b. If the angle between the wires is reduced to $90.0°$, what new force does each wire exert? ($106\ N$)
   c. As the angle between the wires decreases, what happens to the force in the wires?

4. Joe wishes to hang a sign weighing $750\ N$ so that cable A attached to the store makes a $30.0°$ angle as show below. Cable B is attached to an adjoining building. Calculate the necessary tension in cable B. ($433\ N$)

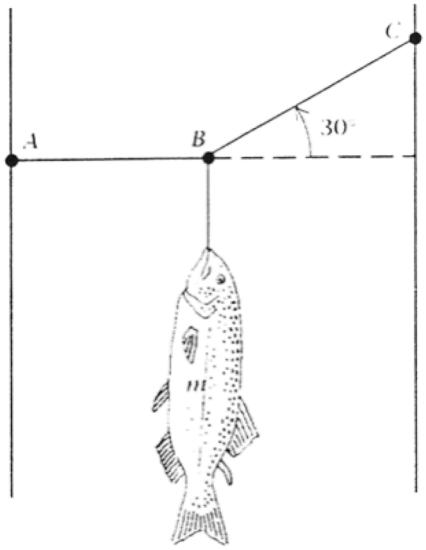


5. A $52.6\ kg$ high-school student hangs from an overhead bar with both hands.
   a. What is the tension in each arm if the bar is gripped with both arms raised vertically overhead? ($258\ N$)
   b. What is the tension in each arm when the arms make an angle of $33°$ with the vertical? ($307\ N$)

6. A plant is hung from wires as shown. What is the tension in each wire if the plant weighs $20.0\ N$? Ignore the weight of the wire. ($T_1 = 16.3\ N$, $T_2 = 18.4\ N$)



7. A fish of mass m is suspended by a string as shown. The string is fastened securely at point C but will pull loose from the wall at A when the string tension exceeds $22\ N$. What is the maximum mass of the fish that can be supported by the string? ($1.30\ kg$)



8. What is the magnitude of the force exerted by the string on the wall at point A above if the suspended fish has mass $m = 0.35\ kg$? ($5.94\ N$)